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Uncertainty analysis of a contact-base multibody model of meshing spur gears

Luca D'Angelo[1,3], **Marco Cirelli**[1,3], **Oliviero Giannini**[1,3]
Alessio Cellupica[2], **Pier Paolo Valentini**[2]

[1]Department of mechanical engineering
University Niccolò Cusano,
Via Don Carlo Gnocchi, 3, 00166, Italy
[luca.dangelo, marco.cirelli, oliviero.giannini]@unicusano.it

[2]Department of enterprise engineering
University of Rome Tor Vergata,
Via del Politecnico 1, 00133, Italy
alessio.cellupica@alumni.uniroma2.eu
valentini@ing.uniroma2.it

[3]ATHENA European University
Advanced Technology Higher
Education Network Alliance

ABSTRACT

The adoption of numerical methods to simulate complex physical phenomena has improved significantly in the last decades thanks to the fast growth of computing hardware and algorithm developments. Numerical methods are based on a simplified physical (and consequently mathematical) model able to reproduce with a certain error the real physical phenomena. The accuracy of the model always depends on the level of approximations adopted and consequently on the uncertain input of the model. Gear dynamics is one of the most discussed topics in engineering because gears are the most common components adopted for power transmission. The methods adopted for gear design can be divided into four families: experiment method (EM), finite element method (FEM), traditional analytical method (AM), and multibody dynamics methods (MUBO). Recently, a novel MUBO model based on contact and having pseudo-rigid teeth (MUBOCO-PR) has been developed. It demonstrates very high accuracy on the evaluation of transmission error in static and dynamic conditions. However, it needs an a priori identification flexibility and contact parameters. In this study, an evaluation of the epistemic uncertainty in parameter identification is performed using the fuzzy arithmetic-based transformation method. The use of this method makes it feasible to accurately assess the contribution of each model parameter's level of uncertainty to the overall degree of uncertainty of the model output. The results, in terms of static TE, are reported and discussed.

Keywords: Gear dynamics, Contact, Multibody model, pseudo-rigid model, epistemic uncertainty.

1 INTRODUCTION

Gear dynamics is one of the most discussed topics in engineering because gears are the most common components adopted for power transmission. For this reason, many researchers are actively working to develop sophisticated models for gear dynamic simulations capable of identifying vibrations generated by teeth impacts, optimizing the design process, and improving the current generation of diagnostic techniques. It has been demonstrated [1] that the variation of mesh stiffness, depending on the number of contact teeth and the flexibility of the gear, becomes a source of vibration in dynamic conditions. A meaningful parameter to evaluate this excitation is the transmission error (TE) that represents the deviation in position of the driven gear and the position it would occupy if the gear drive were perfectly conjugate [2].

$$TE(t) = \theta_1(t) r_{b1} + \theta_2(t) r_{b2} \tag{1}$$

for the i-th gear (i=1,2), is the angular position, measured from the nominal position, and the base radii, respectively. The adoption of numerical methods to simulate complex physical phenomena has improved significantly in recent decades, thanks to the fast growth of computing hardware and algorithm developments. Numerical methods are based on a simplified physical (and consequently mathematical) model able to reproduce with a certain error the real physical phenomena. The accuracy of the model always depends on the level of approximations adopted and consequently on the uncertain input of the model. The methods adopted for gear design can be divided into four families: experiment method (EM), finite element method (FEM), traditional analytical method (AM) [3, 4, 5], and multibody dynamics methods (MUBO). FEs methods are accurate, but they lack generality and are very computationally demanding; AMs are hard to set because they need a complex pre-processing phase especially if the gears have specific profile characteristics or load variability [6, 7]; MUBO represent a good compromise between computational time and reliability. In MUBO the gears are generally considered as rigid bodies and the contact between the involute profiles is established through a detection method. Consistent with the approach in [8], the contact force generated at the contact point is based on a penalty contact force. Thanks to the multibody approach, it is possible to effectively represent the evolution of contact points along the profile, the friction of the surfaces, and it is also possible to assess the dynamic of the system under several operating conditions, such as variable torque or acceleration. In [9] a family of MUBO contact-based models able to consider the compliance of teeth through pseudo-rigid approach (MUBOCO-PR) has been introduced and compared. MUBOCO-PR is essentially a pseudo-rigid multibody system in which the teeth are considered as rigid bodies connected to the main body (the gear foundation) through specific joints located in the dedendum circle and an equivalent spring. In [10, 11] it has been demonstrated the high-accuracy of the MUBOCO-PR in the evaluation of the transmission error both in static and dynamic conditions. As in all simplified models, even the MUBOCO-PR requires an *a priori* identification of the lumped parameters. In [10] a reliable approach to identify these parameters is provided. However, a systematic approach to estimate the uncertainty of the model is not already discussed. Actually, during the modeling process of multibody model simulations, different types of epistemic uncertainty may arise, and they can be categorized according to their origin and their nature. Namely, the uncertainty may be related to the lack of knowledge of the initial or operating conditions. Moreover, subjectivity in implementation, such as the use of various numerical approaches, will have an impact on the outcome, constituting another source of uncertainty. Lastly, but not least, it is crucial to include in the overall uncertainty evaluation the simplifications or idealizations made in the mathematical models to guarantee or facilitate the numerical evaluation. As reported in [12], these uncertainties can be successfully represented and quantified by fuzzy numbers within the context of comprehensive MB modelling, i.e. modelling both the system and potential uncertainties. Following the idea of Zade [13], Hanss [14, 15] proposed a novel method defined "transformation method" to describe the lack of certainty of the input parameters as fuzzy numbers and evaluate its propagation through the model. Thanks to this approach it is possible to provides a simple but very powerful tool for estimating the uncertainties of the output as fuzzy-valued quantities. This method is a very practical application of fuzzy arithmetic to solve complex engineering problems. However, the disadvantage of the transformation method is that it usually requires many physical models to be evaluated. Furthermore, the number of evaluations increases exponentially with the number of uncertain input parameters. In this study, an evaluation of the uncertainty of the MUBOCO-PR model is carried out using the fuzzy arithmetic-based transformation method [13] to evaluate the effects of the epistemic uncertainty of the MUBOCO-PR models.

2 MULTIBODY MODEL DESCRIPTION

MUBOCO-PR is a novel family of multibody gear models. It considers the teeth separated from the rim body and connected with them using a revolute joint placed near the root radius and an equivalent torsion spring, as reported in Figure 1. In this way, when a generic force acts on the teeth, the reaction force is balanced from the revolute constraint while the torsion spring satisfies the rotational equilibrium through the following equation:

$$T_{junction} = -k_{\varphi}(\varphi - \varphi_0)^{n_{k\varphi}} - c_{\varphi}\dot{\varphi}^{n_{c\varphi}} + T_0 \tag{2}$$

where k_φ and c_φ are the spring stiffness and damping coefficients, respectively; $n_{k\varphi}$, $n_{c\varphi}$ the exponents of the relative rotation and angular velocity, respectively; T_0 the free length spring torque; φ the angle of the current rotation; φ_0 the free angle. Assuming T_0=0 and φ_0=0, only four parameters, namely k_φ, c_φ, $n_{k\varphi}$, $n_{c\varphi}$, need to be identified. This approach makes it possible to consider the gear as a pseudo-rigid multibody model. The interaction between two or more gears occurs through the pseudo-rigid teeth.

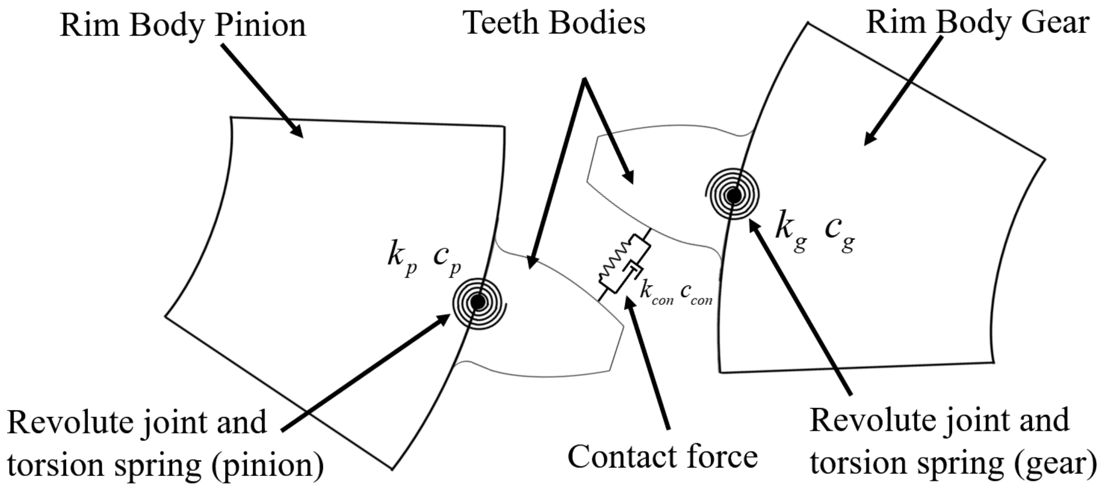

Figure 1. *Schematic representation of MUBOCO-PR*

A detection method based on two steps (general detection and detailed detection) is adopted to identify which teeth must be considered for the contact [16]. Once two teeth enter in contact and the involute profiles overlap, a restitution force normal to the two profiles, is generated depending on the penetration and the penetration speed. The following equation describes how it is computed the normal restitution force:

$$F_n = k_{con}\delta^{m_1} + c_{con}\frac{\dot{\delta}}{|\dot{\delta}|}|\dot{\delta}|^{m_2}\,\delta^{m_3} \tag{3}$$

where δ is the penetration, $\dot{\delta}$ is the penetration speed, k_{con} and c_{con} are the stiffness and the damping coefficients, respectively, m_1, m_2 and m_3 are the stiffness, the damping, and the indentation exponents, respectively. Considering this approach, we have four parameters able to influence the stiffness (and so quasi-static effects) and five parameters able to influence the damping (and so dynamic effects). In this paper, fuzzy logic is adopted for the stiffness coefficients only.

3 EPISTEMIC UNCERTAINTY MODEL DESCRIPTION

Fuzzy logic is, generically, a way to model logical reasoning where the truth of a statement is not a binary true or false like it is with classical (crisp) logic, but rather it is a degree of truth that ranges from zero which is absolutely false to the absolutely true one. Actually, classical set theory reaches its limits when the property that determines the membership of an element to a set is defined so that a clear belonging definition to that set is no longer possible. Forming a special class of fuzzy sets, the fuzzy numbers were defined and a new arithmetic based on their definition was created.

Using fuzzy numbers to represent the input parameter of a mathematical model is a practical way to model the uncertainties inherent to the parameters. Each model parameter, described as a fuzzy number, is characterized by a membership function $\mu(x)$, $0 \leq \mu(x) \leq 1$, which reflect the degree of uncertain of the numerical quantification. The transformation method provides the implementation of fuzzy arithmetic and can be used to evaluate systems with fuzzy-valued parameters. The method is generally available in a reduced or general form, depending on the modality of consideration of fuzzy input values.

In this case, where the general form is implemented, the main steps are reported in the flow diagram in Figure 2 and can be described as follows. The uncertain system is characterized by N_p fuzzy-valued model parameters (l_i, i=1, …, N_p) defined as shown in Figure 3. The first step of the method is the discretization of the inputs in a series of intervals assigned to the levels μ_j (j=1, …, N_c) of membership. As a result, the input is subdivided in a range of membership [0,1] by equally spaced $\Delta\mu=1/N_c$ gaps. At every membership cut, the intervals contain the first $a^{(j)}$ and the last $b^{(j)}$ value of the i-th l_i parameter, and N_s-2 linearly spaced values between these limits. In the second step, input intervals values of each parameter are combined at each membership cut, obtaining N_c arrays of N_{sim} elements. Here N_{sim} is calculated as:

$$N_{sim} = \bigcup_{i=1}^{N_p} N_{s,i} \tag{4}$$

Each array element represents a specific sample of possible parameter combinations and can be used as a set of input parameters for the model to be evaluated. Therefore, N_{sim} results of the model calculation are obtained and stored in N_c arrays of N_{sim} elements for each membership level. Then, all the arrays are sorted and transformed in the output intervals $O^{(j)}=[a^{(j)}, b^{(j)}]$ at each membership level μ_j. The last step requires the aggregation of the output intervals constituting the fuzzification of the system result o.

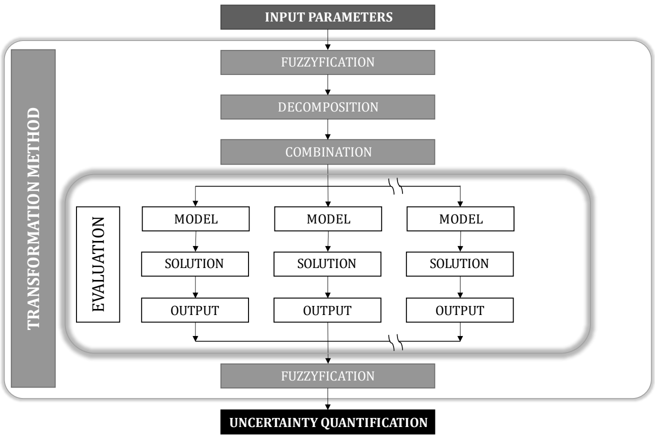

Figure 2. *Flow chart of the transformation method.*

In addition, an analytical process is implemented after the model evaluation to quantify the effects of each fuzzy-valued input parameter l_i on the overall fuzzy model output o. A sensitivity factor, ρ_i, was defined for each input parameter to express the effect of its uncertainty on the overall uncertainty of the model output at each membership level μ_j or averaged across all the levels.

The transformation method considers the model as a black box. This is a great advantage because it is possible to reduce fuzzy arithmetic to multiple crisp-number operations made on both the inputs and the outputs of the model simulation. For this reason, its implementation in existing software environments is relatively straightforward, and its area of application is not subject to restriction.

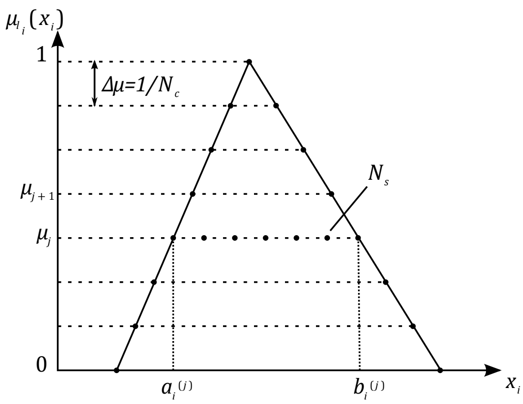

Figure 3. *A triangular fuzzy number representation.*

4 NUMERICAL EXAMPLE

The fuzzy-based transformation method is applied to a specific gear set to identify the epistemic uncertainty of the static transmission error (STE) calculation using the MUBOCO-PR model (*Table 1*). The input parameters that mainly influence the STE result are the two main stiffnesses:

- the torsional stiffness of the torsion spring;
- the contact stiffness.

Cirelli et al. [10] reported that the stiffness of the torsion spring changes along the contact evolution. It is caused by several factors: the distance variation from the contact point and the position of the joint; the variation of the relative angle between the load direction and the tooth axis. Furthermore, because the contact stiffness depends on the curvature of the involute profile, even the contact stiffness is influenced by the position of the contact point. As a general rule, in MUBOCO-PR the values have been assumed constants and considered as the point of contact is on the working pitch circle. This choice agrees with the STE obtained with FEM simulations of a collection of meshing gears. However, it is significant to evaluate the influence of this appropriate choice on the model output by applying the transformation method.

Table 1. *Geometrical properties of adopted gears*

	Value	Nomenclature	UoM
m	3	Module	mm
Z	50	Number of Teeth	
d_p	150	Pitch Circle Diameter	mm
ϕ_p	20	Pressure angle	deg
d_b	140.95	Base Circle Diameter	mm
d_{tip}	156	Tip Circle	mm
d_{root}	141	Root Circle	mm
t	20	Tooth Width	mm
R_{fill}	0.75	Fillet radius	mm
b	0.07	Backlash	mm

Therefore, two fuzzy triangular numbers were created, adopting the analytically defined modal value and the definition of the support limits. Support for a fuzzy number is defined as the base interval of the number. In other words, it represents the set of values at the zero level of the membership function thus the inherent total uncertainty of the parameter.

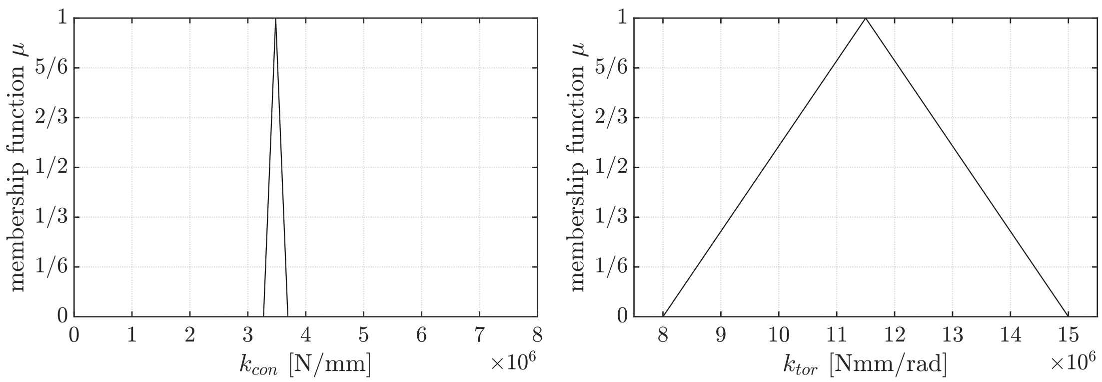

Figure 4. *The two fuzzy input parameters. On the left: the contact stiffness is shown. On the right: the torsion stiffness.*

The two input parameters for the fuzzy-based transformation method are shown in Figure 3. Considering the same scale for both plots, it can be seen that the support adopted for the contact stiffness is smaller than the support of the torsional spring stiffness. The dimension of the support depends on the data uncertainty level. For the contact stiffness computed with the Weber model [17] a limited range of variability has been recorded during the contact evolution. This leads to a limited range of support for contact stiffness. Conversely, the torsional stiffness value that introduces a relevant approximation in the model is defined with relatively larger support due to its changing along the meshing line.

As Figure 4 shows, the selected membership function is a linear trend that increases as the limits of the set considered for each membership value decrease. After the definition of the two parameters as fuzzy numbers, following the transformation, the number of cuts (N_c) and the number of values to sample for each cut (N_s) were defined as reported in Table 2.

Table 2. *Properties of fuzzy input parameters.*

Parameter	Modal value	Support	Dimensions	N_c	N_s
Rotational spring stiffness	1.15E+07	[0.80, 1.50]E+07	Nmm/rad	6	7
Contact stiffness	3.48E+06	[3.27, 3.69]E+07	N/mm	6	7

Therefore, the combination at each cut is carried out, resulting in 49 simulation input sets to be evaluated for each level of membership.

In order to evaluate the STE through the multibody simulations, an increasing rotational velocity and torque are imposed on the pinion and gear respectively. The velocity applied to the center revolute of the pinion follows a cubic ramp with the following algebraic expression:

$$\omega_p(t) = step(t, t_i, \omega_i, t_f, \omega_f), \quad k = 1,2,3 \tag{5}$$

where t_i, t_f, ω_i and ω_f are the initial and final time and velocity values, respectively, and the function *step*(x ,x_0, h_0, x_1, h_1) has the following algebraic expression:

$$step = \begin{cases} h_0, & x \leq x_0 \\ h_0 + (h_1 - h_0)\left[3\left(\dfrac{x - x_0}{x_1 - x_0}\right)^2 - 2\left(\dfrac{x - x_0}{x_1 - x_0}\right)^3\right], & x_0 < x < x_1 \\ h_1, & x \geq x_1 \end{cases} \tag{6}$$

The maximum rotational velocity value is set at 0.1 rad/s. After the ramp, the velocity value remains constant for a period of time sufficient to eliminate any transient effect. The same procedure and ramp is applied for the imposed torque on the meshing gear. The stationary value, for the case here proposed, is set at 100 Nm.

The result of each simulation is the STE along one meshing cycle. This STE output is selected in the stationary end phase of the simulation. Therefore, the output result is not only a single fuzzy number but a fuzzy function that varies along the meshing period. The variation of the value and the shape of the STE function with the changing of the stiffnesses can be evaluated. Moreover, all the results are compared with the quasi-static FEM results.

As a reference model, a finite element 2D model in-plane strain condition is adopted. 2D modeling is necessary to save time calculations and it is an appropriate way to represent the physics of contact if used with awareness. Due to the large face width of the gears selected, a plane strain condition is suitable for this study. The elements adopted for 2D analysis are quadratic. Two revolute constraints are kinematic coupled with the hubs of the gears and a high mesh refinement is imposed in the contact region. The FEM results agree with the MUBOCO-PR solution with the two stiffness values set as the analytically defined modal value, as already presented in [10].

To systematically evaluate the sensitivity of the model output with respect to the fuzzy interval of uncertainty, the influence index is defined according with [14]. Its characterization lies on the calculation of the components of the gradient in each parameter direction. This computation must be performed at each level of the membership function. Thus, a series of different domains around the modal value are investigated and all the combinations at each membership level of the input parameters and the corresponding output are considered. For each membership level, it is possible to determine the g vector of N_p elements, as the value obtained by summing the absolute value of the partial derivative in each i-th input parameter.

$$g_j = \left[\sum \left| \frac{\partial o}{\partial l_1} \right| \quad \sum \left| \frac{\partial o}{\partial l_2} \right| \quad \cdots \quad \sum \left| \frac{\partial o}{\partial l_{N_p}} \right| \right] \tag{7}$$

Consequently, a single value is computed for each parameter, considering all the membership levels. This value was evaluated through the calculation of the mean value of all the i-th elements of g_j weighted with the level of membership.

$$k_i = \frac{1}{Ncut} \sum_{j=1}^{Ncut} j \; g_j(i) \;\; \forall i = 1,..,Np \text{ parameter} \tag{8}$$

The final sensitivity percentage is calculated as the ratio between the single parameter and the sum of all k_i values.

$$\rho_i = \frac{k_i}{\sum_{i=1}^{Np} k_i} \;\; \forall i = 1,..,Np \text{ parameter} \tag{9}$$

5 RESULTS DISCUSSION

The results from the model are summarized in Figure 5. The STE is shown throughout the mesh cycle, computed for each input combination. The FEM result is represented as a black line with cross marks, while, the series of crisp results obtained with the model simulation are highlighted with a continuous grayscale line that becomes lighter as the membership level decreases.

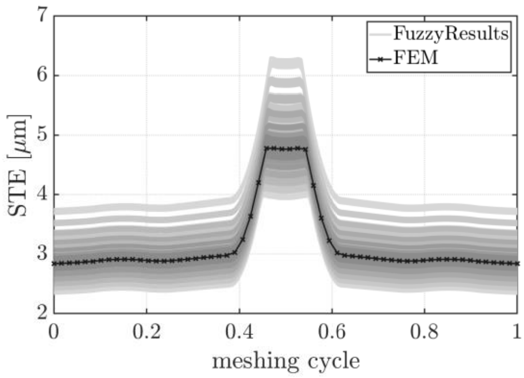

Figure 5. *STE.vs. meshing cycle for each input combination.*

Therefore, it is possible to create a fuzzy STE output at each meshing cycle position through the fuzzification process.

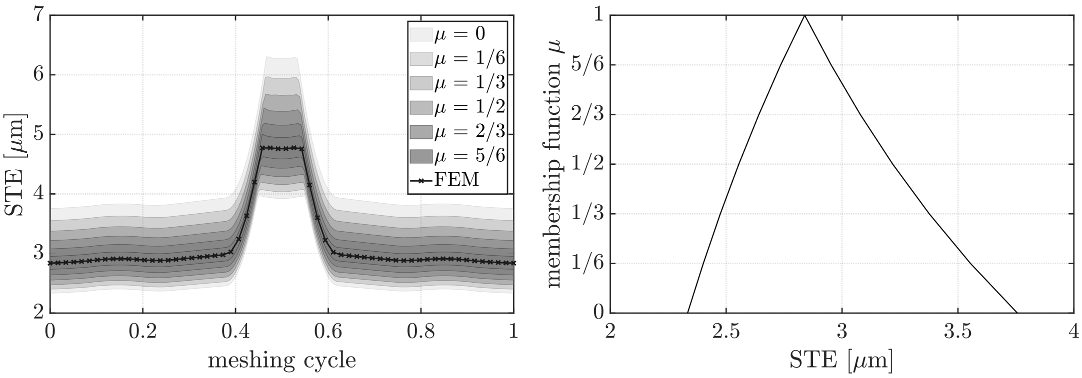

Figure 6. *Fuzzificated STE output along the meshing cycle (right). Particular fuzzy STE output at 0.2 meshing cycle ratio (left)*

Although the input parameters were defined as triangular fuzzy numbers, Figure 6 shows that the output STE, calculated through the MUBOCO-PR model, does not have the same membership shape. This reveals the nonlinearity of the system output with respect to the two stiffnesses. In particular, the decrease in the stiffness values increases the STE more than linearly. On the other hand, the output decreases less than linearly while the stiffnesses increase.

Moreover, the influence of each input on the global output is performed by applying the fuzzy-based transformation method. The sensitivity ratio ρ_i was first evaluated for the two input parameters at each membership level.

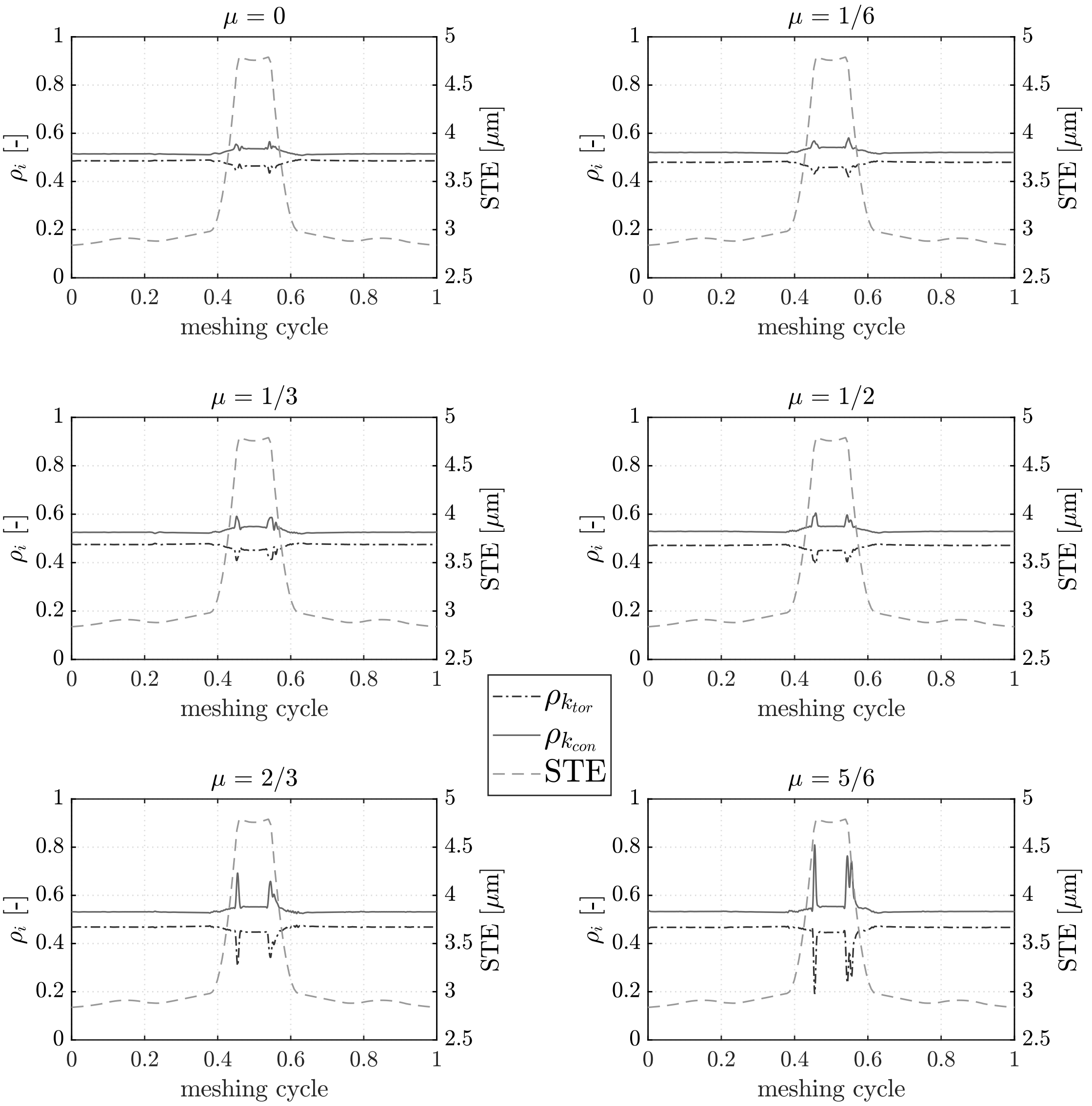

Figure 7. *Sensitivity ratios at each membership level along the meshing cycle period.*

Figure 7 shows the evolution of the sensitivity ratios at each membership level before the mean value k_i is calculated. The ratios at each α-cut appears to maintain some key characteristic. Indeed, the sensitivity ratios of the two input parameters during the two-tooth meshing phase are constantly close to 0.5, which means they have almost the same influence on the STE uncertainty. Furthermore, the sensitivity ratio of the contact stiffness is continuously larger than the value of the torsional spring stiffness. However, it is possible to highlight that, with the increase in the membership level, there is a reduction in the overall uncertainty, leading to a decrease of the torsional spring stiffness sensitivity ratio with respect to the contact stiffness one. This phenomenon is related with the fact that the supports, and thus the level of total uncertainty selected for the two input parameters, are very different in amplitude. For this reason, at a low membership level, the sensitivity ratio of the torsional spring has a greater influence than at a high level. On the other hand, considering the high membership cuts, where the interval of uncertainty decreases towards the modal value, the importance of the contact sensitivity ratio increases.

Furthermore, at every membership level, the nearly constant trend of the sensitivity ratios diverges during the transition phase in the number of teeth in contact. In particular, passing from two to one couple of teeth, the contact stiffness ratio gently rises and peaks in the region of the detachment of the second pair of teeth. During the one-tooth phase, the ratio returns at a value slightly higher than the previous phase but maintains the same constant trend. Then, with the engagement of the second couple of teeth, the sensitivity ratio returns to a value near 0.5. The presence of a double peak during the engagement of the second pair of teeth is due to the intrinsic asymmetry of the gear dynamics. Namely, the possible different behavior in the access and recess of the pair of teeth mating.

Finally, the ratios at each membership level are averaged with the mean value weighted by the membership-level fraction. Figure 8 shows the overall influence indexes along the meshing cycle. Due to the presence of the weights in equation (8), the last membership cuts have more effect on the overall ratios evolution along the meshing period. However, all the membership levels affect its value, resulting in a comprehensive influence indexes related to the uncertainty of the inputs. The evolution of the index along the meshing cycle is characterized by the same main features of the ratios at every membership level. Namely, the fact that the influence index associated at the contact stiffness is constantly higher than the one related to the torsional spring stiffness. Moreover, the presence of the two peaks during the change in the number teeth mating can be still observed while the indexes remain constant in the other phases.

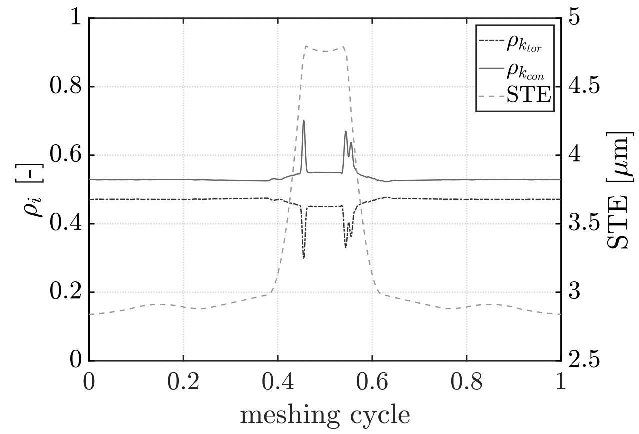

Figure 8. *Final sensitivity ratio of the two inputs along the meshing cycle*

6 CONCLUSIONS

In this study, the fuzzy arithmetic-based transformation method is used to assess the epistemic uncertainty in the parameters of a multibody model contact based with pseudo-rigid teeth. The torsional stiffnesses between each tooth and the gear body and the contact stiffness are considered as fuzzy-valued input parameters. Thanks to this approach, it is shown possible to determine with consistency how much the level of uncertainty of each model parameter contributes to the overall level of uncertainty of the model output. The results, in terms of static transmission error, computed along the meshing cycle, confirm the expected nonlinearity of the model output with respect to the two inputs selected. Furthermore, it is shown that a linear decrease of the two stiffnesses is related to a more than linear increase in the STE. Owing to the fuzzy-based method adopted in this study, it was also possible to show that, although the uncertainty of the torsional stiffness is larger than the contact stiffness, the model sensitivity is almost equally influenced by the variation of the two inputs considered. Moreover, it is verified that the contact stiffness has a slightly more significant influence on the STE results at a high membership level or, in other words, near the analytically defined modal values. The relevant results demonstrate the potentiality of the method and put the basis for a more complex uncertainty evaluation of this type of dynamic gear model.

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